Instructions. (30 points) Solve each of the following problems.

(1^{pts}) **1.** $\lim_{x\to -3} \frac{x^2-9}{x-3} =$

(a) -6 (✔b) 0

(c) Does Not Exist (d) 6

Solution:

$$\begin{aligned}\lim_{x\to -3} \frac{x^2-9}{x-3} &= \frac{(-3)^2-9}{(-3)-3} \\ &= \frac{9-9}{-6} \\ &= \frac{0}{-6} = 0\end{aligned}$$

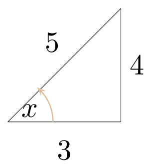
(1^{pts}) **2.** If $\cos x = \frac{3}{5}$, $\frac{3\pi}{2} < x < 2\pi$ then $\tan x =$

(a) $\frac{-4}{5}$ (✔b) $\frac{-4}{3}$

(c) $\frac{-5}{4}$ (d) $\frac{-3}{4}$

Solution:

Since $\cos x = \frac{3}{5} = \frac{\text{adjacent}}{\text{hypotenuse}}$ we draw a triangle is shown below: Now, $H^2 = A^2 + O^2 \Rightarrow 25 = 9 + O^2 \Rightarrow O = 4$. Since $\frac{3\pi}{2} < x < 2\pi$, then x lies in the forth quadrant. Hence since the $y-$ axis is negative, then $\sin x < 0$ and since the $x-$axis is positive then $\cos x > 0$. Therefore $\tan x < 0$ Hence $\tan x = \frac{-4}{3}$.



(1^{pts}) **3.** $\lim_{x\to -\infty} \frac{2x^2+1}{6+x^2-3x^3} =$

(✔a) 0 (b) $\frac{-2}{3}$

(c) $\frac{2}{3}$ (d) $-\infty$

Solution:

Since $\lim_{x\to-\infty}(2x^2+1) = \infty = \lim_{x\to-\infty}(6+x^2-3x^3)$ we have I.F. type ∞/∞. Divide each term in the numerator and each term in the denominator by the highest power in the denominator.

$$\begin{aligned}\lim_{x\to-\infty}\frac{2x^2+1}{6+x^2-3x^3} &= \lim_{x\to-\infty}\frac{\frac{2x^2+1}{x^3}}{\frac{6+x^2-3x^3}{x^3}}\\ &= \lim_{x\to-\infty}\frac{\frac{2x^2}{x^3}+\frac{1}{x^3}}{\frac{6}{x^3}+\frac{x^2}{x^3}-\frac{3x^3}{x^3}}\\ &= \lim_{x\to-\infty}\frac{\frac{2}{x}+\frac{1}{x^3}}{\frac{6}{x^3}+\frac{1}{x}-3}\\ &= \frac{0+0}{0+0-3}=0\end{aligned}$$

(1[pts]) **4.** $\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right)+\sin\left(\frac{\pi}{9}\right)\cos\left(\frac{\pi}{18}\right)=$

(a) 2 (b) $\frac{\sqrt{3}}{2}$

(c) 0 (✔d) $\frac{1}{2}$

Solution:

$$\begin{aligned}\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right)+\sin\left(\frac{\pi}{9}\right)\cos\left(\frac{\pi}{18}\right) &= \sin\left(\frac{\pi}{18}+\frac{\pi}{9}\right)\\ &= \sin\left(\frac{\pi}{6}\right)\\ &= \frac{1}{2}.\end{aligned}$$

(1[pts]) **5.** The curve $f(x)=\dfrac{x^2+2x-3}{x^3-9x}$ has a vertical asymptote at

(✔a) $x=0, \quad x=3$ (b) $x=0, \quad x=\pm 3$

(c) $x=0, \quad x=-3$ (d) $y=0, \quad y=3$

Solution:

Write $f(x)=\dfrac{(x+3)(x-1)}{x(x-3)(x+3)}$. The zeroes of the denominator are -3, 0, and 3. To check that $x=-3$ is a vertical asymptote or not we take the limit at -3 from both sides. $\lim_{x\to-3}f(x)=\lim_{x\to-3}\dfrac{\cancel{(x+3)}(x-1)}{x\cancel{(x+3)}(x-3)}=\lim_{x\to-3}\dfrac{x-1}{x(x-3)}=\dfrac{-4}{18}=\dfrac{-2}{9}$. Hence $x=-3$ is not a vertical asymptote. To check that $x=3$ we take the limit $\lim_{x\to3^+}f(x)=\lim_{x\to3^+}\dfrac{(x+3)\overset{+}{(x-1)}}{x\underset{+}{(x-3)}(x+3)}=\infty$. Hence $x=3$ is a vertical asymptote. To check that $x=0$ we take the limit $\lim_{x\to0^+}f(x)=\lim_{x\to0^+}\dfrac{(x+3)\overset{-}{(x-1)}}{x\underset{-}{(x-3)}(x+3)}=\infty$. Hence $x=0$ is a vertical asymptote.

Thus the function has vertical asymptote at $x=0$, and $x=3$.

(1pts) **6.** If $f(x) = \begin{cases} x+1, & \text{if } x < -2; \\ 1, & \text{if } x = -2; \\ \dfrac{x^3+8}{x^2-4}, & \text{if } x > -2. \end{cases}$ Then $\lim\limits_{x \to -2^+} f(x) =$

(✔) -3 (b) -1

(c) 3 (d) 0

Solution:

Note that when computing limit as $x \to a^+$ means you approaches a from the right side that is $x > a$.

$$\begin{aligned} \lim_{x\to -2^+} f(x) &= \lim_{x\to -2^+} \frac{x^3+8}{x^2-4} \\ &= \lim_{x\to -2^+} \frac{x^3+8}{x^2-4} && \text{A direct substation will give us I.F. } 0/0 \\ &= \lim_{x\to -2^+} \frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)} && \text{Factoring } x+2 \text{ from denominator and numerator} \\ &= \lim_{x\to -2^+} \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}(x-2)} \\ &= \lim_{x\to -2^+} \frac{x^2-2x+4}{x-2} \\ &= \frac{(-2)^2-2(-2)+4}{-2-2} = \frac{12}{-4} = -3. \end{aligned}$$

(1pts) **7.** $\dfrac{d^{37}}{dx^{37}}(\cos x) =$

(✔) $-\sin x$ (b) $\sin x$

(c) $\cos x$ (d) $-\cos x$

Solution:

Since $37 = 4(9)+1$ then $\dfrac{d^{37}}{dx^{37}}(\cos x) = \dfrac{d}{dx}(\cos x) = -\sin x$.

(1pts) **8.** If $3x \le f(x) \le \dfrac{x^2+x-2}{x-1}$, $x \in [0,2], x \ne 1$, then $\lim\limits_{x\to 1} f(x) =$

(a) -3 (✔) 3

(c) 1 (d) 0

Solution:

Since $\lim\limits_{x\to 1}(3x) = 3$ and $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x-1} = \lim\limits_{x\to 1} \dfrac{(x-1)(x+2)}{x-1} = \lim\limits_{x\to 1}(x+2) = 3$, then by The Sandwich Theorem we have $\lim\limits_{x\to 1} f(x) = 3$.

(1pts) **9.** $\lim\limits_{x\to 0} \dfrac{x^2}{\sin^2(5x)} =$

(a) 25 (✔) $\dfrac{1}{25}$

(c) $\dfrac{1}{5}$ (d) 5

Solution:

$$\begin{aligned}
\lim_{x\to 0}\frac{x^2}{\sin^2(5x)} &= \lim_{x\to 0}\left(\frac{x}{\sin(5x)}\right)^2 && \tfrac{a^n}{b^n}=\left(\tfrac{a}{b}\right)^n.\\
&= \lim_{x\to 0}\left(\frac{5x}{5\sin(5x)}\right)^2 && \text{make the top similar to the angle}\\
&= \left(\frac{1}{5}\lim_{x\to 0}\frac{5x}{\sin(5x)}\right)^2 && \text{Use that } \lim_{x\to 0}\frac{\sin x}{x}=1=\lim_{x\to 0}\frac{x}{\sin x}\\
&= \left(\frac{1}{5}.1.\right)^2=\frac{1}{25}
\end{aligned}$$

(1pts) **10.** $\lim_{y\to 0} 6y^2\cot y\csc(2y) =$

(a) $\frac{1}{3}$ (b) -3

(✔) 3 (d) 1

Solution:

Direct substation will give us I.F. type 0/0.

$$\begin{aligned}
\lim_{y\to 0} 6y^2\cot y\csc(2y) &= \lim_{y\to 0}\frac{6y^2}{\tan y\sin(2y)}\\
&= 3\lim_{y\to 0}\frac{y}{\tan y}\frac{2y}{\sin(2y)} \qquad \text{use } \lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1=\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\\
&= 3(1)(1)=3
\end{aligned}$$

(1pts) **11.** If $y=\frac{\sin x}{1+\cos x}$, then $y' =$

(a) $\frac{\sin x}{1+\cos x}$ (b) $\frac{\cos x}{(1+\cos x)^2}$

(✔) $\frac{1}{1+\cos x}$ (d) $\frac{\cos x}{1+\cos x}$

Solution:

$$\begin{aligned}
y' &= \frac{(1+\cos x)(\cos x)-(\sin x)(-\sin x)}{(1+\cos x)^2}\\
&= \frac{\cos x+\cos x\cos x+\sin x\sin x}{(1+\cos x)^2}\\
&= \frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2} && \text{Use the fact } \cos^2 x+\sin^2 x=1\\
&= \frac{1+\cos x}{(1+\cos x)^2} && \text{simplify}\\
&= \frac{1}{1+\cos x}.
\end{aligned}$$

(1^{pts}) **12.** The graph of the function $F(x) = x^3 - 3x$, has a horizontal tangent line at

(a) $x = 0$ (✔) $x = \pm 1$

(c) $x = -1$ (d) $x = 1$

Solution:

$$F(x) = x^3 - 3x$$
$$F'(x) = 3x^2 - 3 = 3(x^2 - 1)$$
$$F'(x) = 0$$
$$3(x^2 - 1) = 0$$
$$x = \pm 1.$$

(1^{pts}) **13.** An equation for the tangent line to the curve $f(x) = x^3 - x$ at $x = -1$ is

(a) $y = -2x - 2$ (b) $y = 2x - 2$

(✔) $y = 2x + 2$ (d) $y = -2x + 2$

Solution:

The slope of the tangent line to $f(x) = x^3 - x$ at $x = -1$ is $f'(-1)$.

$$f(x) = x^3 - x \Rightarrow f'(x) = 3x^2 - 1.$$

Hence

$$\text{the slope of the tangent } = f'(-1) = 3(-1)^2 - 1 = 2.$$

Also $f(-1) = (-1)^3 - (-1) = 0$. Now, we have $m = 2$ and $(-1, 0)$, hence

$$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 2(x + 1) \Rightarrow y = 2x + 2.$$

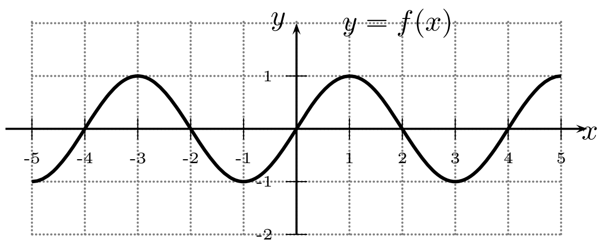
(1^{pts}) **14.** The accompanying figure shows the graph of $y = f(x)$. Then the period of $y = f(x)$ is

(a) 2π

(✔) 4

(c) 3

(d) 2



Solution:

From the graph we can see that the function repeat itself every 4 units. Hence the period is 4

(1^{pts}) **15.** If $\lim\limits_{x\to 1} f(x) = 5$, then must $f(x)$ be defined at $x = 1$.

(a) True

(✓) False

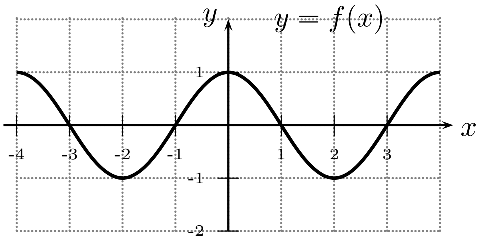
Solution:

False. Let $f(x) = \dfrac{x^2+3x-4}{x-1}$. Then $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{x^2+3x-4}{x-1} = \lim\limits_{x\to 1} \dfrac{(x-1)(x+4)}{x-1} = \lim\limits_{x\to 1}(x+4) = 5$, but $f(1)$ is undefined.

(1^{pts}) **16.** The accompanying figure shows the graph of $y = f(x)$. Then $f'(-1) > f'(1)$.

(✓) True

(b) False

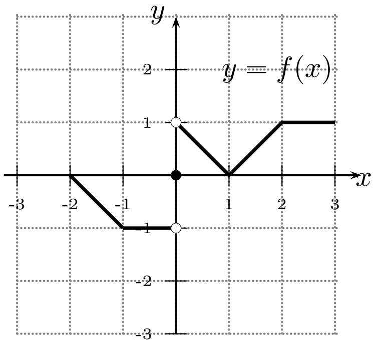


Solution:

From the graph we can see that the tangent line to the graph of $y = f(x)$ at -1 is rising up (left to right) and hence its slope is positive. Thus, since the first derivative is the slope of the tangent line to the graph at -1 then $f'(-1) > 0$.

From the graph we can see that the tangent line to the graph of $y = f(x)$ at 1 is falling down (left to right) and hence its slope is negative. Thus, since the first derivative is the slope of the tangent line to the graph at 1 then $f'(1) < 0$. Now, $f'(-1) > 0 > f'(1)$.

(1^{pts}) **17.** The accompanying figure shows the graph of $y = f(x)$. Then $\lim\limits_{x\to 0} f(x) =$



(a) -1 (✓) Does Not Exist

(c) 1 (d) 0

Solution:

since $\lim\limits_{x\to 0^+} f(x) = 1 \neq -1 = \lim\limits_{x\to 0^-} f(x)$, then $\lim\limits_{x\to 0} f(x)$ does not exist.

(1^{pts}) **18.** $\lim\limits_{x\to 2} \dfrac{3-\sqrt{x+7}}{x-2} =$

(a) $\dfrac{1}{6}$ (✓) $\dfrac{-1}{6}$

(c) 6 (d) 0

Solution:

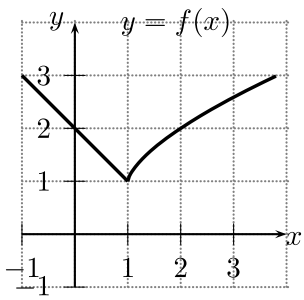
A direct substation will give us I.F. 0/0. Now,

$$\begin{aligned}
\lim_{x\to 2}\frac{3-\sqrt{x+7}}{x-2} &= \lim_{x\to 2}\frac{3-\sqrt{x+7}}{x-2}\cdot\frac{3+\sqrt{x+7}}{3+\sqrt{x+7}} && \text{Multiply by } 1=\frac{3+\sqrt{x+7}}{3+\sqrt{x+7}}\\
&= \lim_{x\to 2}\frac{3^2-(\sqrt{x+7})^2}{(x-2)(3-\sqrt{x+7})}\\
&= \lim_{x\to 2}\frac{9-(x+7)}{(x-2)(3+\sqrt{x+7})} && \text{Simplify}\\
&= \lim_{x\to 2}\frac{-\cancel{(x-2)}}{\cancel{(x-2)}(3+\sqrt{x+7})}\\
&= \lim_{x\to 2}\frac{-1}{3+\sqrt{x+7}}\\
&= \frac{-1}{3+\sqrt{2+7}}\\
&= \frac{-1}{6}.
\end{aligned}$$

(1^{pts}) **19.** The accompanying figure shows the graph of $y = f(x)$. Then f is differentiable at $x = 1$.

(a) True

(✓) False



Solution:

f is not differentiable at $x = 1$ since the derivative from the left of 1 approaches -1 and from the right of 1 approaches ∞.

(1^{pts}) **20.** If $y = x\cos x\sec x$, then $y' =$

(a) $-\sin x\sec x\tan x$

(✓) 1

(c) -1

(d) 0

Solution:

$$\begin{aligned}
y &= x\cos x\sec x\\
y &= x\cos x\frac{1}{\cos x}\\
y &= x\\
y' &= 1.
\end{aligned}$$

(1^{pts}) **21.** The function $f(x) = \sqrt{|x| - 3}$ is continuous on

(a) $(-\infty, 3) \cup (3, \infty)$

(✔) $(-\infty, -3] \cup [3, \infty)$

(c) $[-3, 3]$

(d) $(-\infty, -3]$

Solution: Notice that $f(x) = \sqrt{|x| - 3}$ is an even root function, then it is continuous on its domain which is the set of real number such that $|x| - 3 \geq 0$. Now

$$\begin{aligned} |x| - 3 \geq 0 &\Leftrightarrow |x| \geq 3 \\ &\Leftrightarrow x \geq 3 \text{ or } x \leq -3. \end{aligned}$$

Hence f is continuous on $(-\infty, -3] \cup [3, \infty)$.

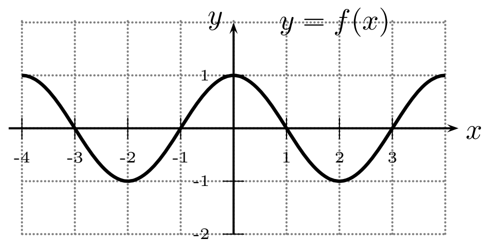
(1^{pts}) **22.** The accompanying figure shows the graph of $y = f(x)$. Then $f'(0) =$

(a) 1

(b) Undefined

(✔) 0

(d) -1



Solution:

From the graph we can see that the tangent line to the graph of $y = f(x)$ at 0 is horizontal ($y = 1$) and hence its slope is zero. Now, since the first derivative is the slope of the tangent line to the graph at 0 then $f'(0) = 0$.

(1^{pts}) **23.** If $y = x^2 - \dfrac{1}{x^3} + \sqrt{5}$, then $y''' =$

(a) $\dfrac{60}{x^6} + \dfrac{1}{2\sqrt{5}}$

(b) $\dfrac{-60}{x^6} - \dfrac{1}{2\sqrt{5}}$

(✔) $\dfrac{60}{x^6}$

(d) $\dfrac{-60}{x^6}$

Solution:

$$\begin{aligned} y &= x^2 - \frac{1}{x^3} + \sqrt{5} \\ y &= x^2 - x^{-3} + \sqrt{5} \\ y' &= 2x + 3x^{-4} \\ y'' &= 2 - 12x^{-5} \\ y''' &= 60x^{-6} = \frac{60}{x^6}. \end{aligned}$$

(1^{pts}) **24.** If $y = \dfrac{3x - 1}{x + 1}$, then $y' =$

(✔) $\dfrac{4}{(x+1)^2}$

(b) $\dfrac{2}{(x+1)^2}$

(c) $\dfrac{-2}{(x+1)^2}$

(d) $\dfrac{6x+2}{(x+1)^2}$

Solution:

$$y' = \frac{\overbrace{(x+1)}^{\text{Bottom}}\ \overbrace{(3x-1)'}^{\text{Derivative of Top}} - \overbrace{(3x-1)}^{\text{Top}}\ \overbrace{(x+1)'}^{\text{Derivative of Bottom}}}{\underbrace{(x+1)^2}_{}^{} \overbrace{}^{\text{Bottom}^2}}$$

$$= \frac{(x+1)(3)-(3x-1)(1)}{(x+1)^2}$$
$$= \frac{3x+3-(3x-1)}{(x+1)^2}$$
$$= \frac{3x+3-3x+1)}{(x+1)^2}$$
$$= \frac{4}{(x+1)^2}.$$

(1^{pts}) **25.** $\lim\limits_{\theta\to 1} \sin\left(\dfrac{\pi\theta}{2\cos(\pi\theta)}\right) =$

(a) 0

(✔) -1

(c) ∞

(d) 1

Solution:

$$\lim_{\theta\to 1}\sin\left(\frac{\pi\theta}{2\cos(\pi\theta)}\right) = \sin\left(\lim_{\theta\to 1}\frac{\pi\theta}{2\cos(\pi\theta)}\right)$$
$$= \sin\left(\frac{\pi(1)}{2\cos\pi}\right)$$
$$= \sin\left(\frac{\pi}{-2}\right)$$
$$= -\sin\left(\frac{\pi}{2}\right)$$
$$= -1.$$

(1^{pts}) **26.** If $y = x\cos x$, then $y' =$

(a) $-\sin x$

(b) $\cos x + x\sin x$

(c) $\cos x + 1$

(✔) $\cos x - x\sin x$

Solution:

$$y' = (x)'\cos x + x(\cos x)'$$
$$= \cos x + x(-\sin x)$$
$$= \cos x - x\sin x.$$

(1[pts]) **27.** $\lim_{x \to 5} \dfrac{x^2 - 25}{x^2 - 3x - 10} =$

(a) $\dfrac{-10}{7}$

(b) ✓ $\dfrac{10}{7}$

(c) $\dfrac{7}{10}$

(d) 0

Solution:

$$\begin{aligned}
\lim_{x \to 5} \frac{x^2 - 25}{x^2 - 3x - 10} &= \lim_{x \to 5} \frac{x^2 - 25}{x^2 - 3x - 10} && \text{A direct substation will give us I.F. } 0/0 \\
&= \lim_{x \to 5} \frac{(x+5)(x-5)}{(x-5)(x+2)} && \text{Factoring } x - 5 \text{ from denominator and numerator} \\
&= \lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}(x+2)} \\
&= \lim_{x \to 5} \frac{x+5}{x+2} \\
&= \frac{5+5}{5+2} = \frac{10}{7}.
\end{aligned}$$

(1[pts]) **28.** $\lim_{x \to 2^+} \dfrac{1 - x^2}{3x - 6} =$

(a) 0

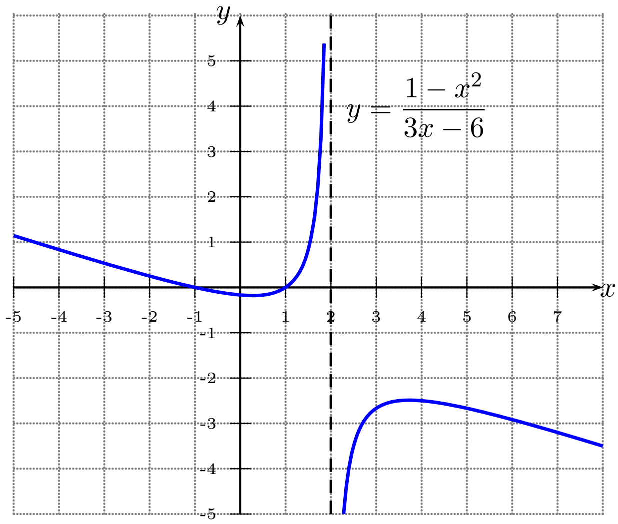
(b) ✓ $-\infty$

(c) ∞

(d) Does Not Exist

Solution:

$$\begin{aligned}
\lim_{x \to 2^+} \frac{1 - x^2}{3x - 6} &= \lim_{x \to 2^+} \frac{1 - x^2}{3x - 6} && \text{A direct substation gives I.F. } -3/0. \\
&= \lim_{x \to 2^+} \frac{1 - x^2}{3x - 6} && \text{If } x > 2 \text{ and near 2,then } 1 - x^2 < 0,\ 3x - 6 > 0. \\
&= \lim_{x \to 2^+} \frac{\overset{-}{1 - x^2}}{\overset{+}{3x - 6}} = -\infty
\end{aligned}$$



(1[pts]) **29.** The curve $f(x) = \dfrac{6x^3 - x + 1}{3x^3 - 4x + 1}$ has a horizontal asymptote at

(✔a) $y = 2$ (b) $y = 0$

(c) $y = 1$ (d) $x = 2$

Solution:

To find the horizontal asymptote we take the limit as $x \to \pm\infty$. Note that both the numerator and the denominator $x \to \pm\infty$, as $x \to \pm\infty$. To find this limit divided both the numerator and the denominator by the highest power of x in the denominator which is x^3. So,

$$\begin{aligned}\lim_{x\to\pm\infty} \frac{6x^3 - x + 1}{3x^3 - 4x + 1} &= \lim_{x\to\pm\infty} \frac{\dfrac{6x^3 - x + 1}{x^3}}{\dfrac{3x^3 - 4x + 1}{x^3}} \\ &= \lim_{x\to\pm\infty} \frac{6 - \dfrac{1}{x^2} + \dfrac{1}{x^3}}{3 - \dfrac{4}{x^2} + \dfrac{1}{x^3}} \\ &= \frac{6 - 0 + 0}{3 - 0 + 0} = 2.\end{aligned}$$

Therefore $y = 2$ is a horizontal asymptote.

(1[pts]) **30.** $\lim_{x\to 0} \dfrac{|x-2| - 2}{x} =$

(a) 0 (b) 1

(c) Does Not Exist (✔d) -1

Solution:

A direct substation gives I.F. 0/0. Note that if $x > 0$ or $x < 0$ near(close) to 0 then $x - 2 < 0 \Rightarrow |x - 2| = -(x - 2)$.

$$\begin{aligned}\lim_{x\to 0} \frac{|x-2| - 2}{x} &= \lim_{x\to 0} \frac{-(x-2) - 2}{x} \\ &= \lim_{x\to 0} \frac{-x + 2 - 2}{x} \\ &= \lim_{x\to 0} \frac{-x}{x} \\ &= \lim_{x\to 0} (-1) \\ &= -1\end{aligned}$$