Workshop Solutions to Sections 1.1 and 1.2

1) $\{x \in \mathbb{R} | -3 \leq x \leq 3\} = [-3,3]$

2) $\{x \in \mathbb{R} | -2 < x < 5\} = (-2,5)$

3) $\{x \in \mathbb{R} | -2 < x \leq 5\} = (-2,5]$

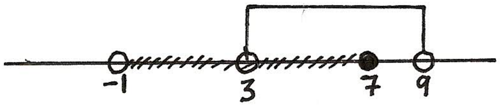
4) $\{x \in \mathbb{R} | -3 \leq x < 3\} = [-3,3)$

5) $\{x \in \mathbb{R} | x \leq -2\} = (-\infty,-2]$

6) $\{x \in \mathbb{R} | x > -2\} = (-2,\infty)$

7) $(-1,7] \setminus (3,9) =$

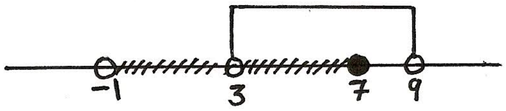
Solution:



$(-1,7] \setminus (3,9) = (-1,3] = \{x \in \mathbb{R} | -1 < x \leq 3\}$

8) $(-1,7] \cup (3,9) =$

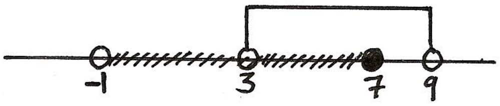
Solution:



$(-1,7] \cup (3,9) = (-1,9) = \{x \in \mathbb{R} | -1 < x < 9\}$

9) $(-1,7] \cap (3,9) =$

Solution:



$(-1,7] \cap (3,9) = (3,7] = \{x \in \mathbb{R} | 3 < x \leq 7\}$

10) $|-7.2| = -(-7.2) = 7.2$

11) $|0.14 - \pi| = |0.14 - 3.14| = |-3| = 3$

OR

$|0.14 - \pi| = -(0.14 - \pi) = \pi - 0.14$

12) $|2 - \pi| = -(2 - \pi) = \pi - 2$

13) $|\pi - 2| = \pi - 2$

14) The solution of the inequality $-3x + 5 < -13$ is

Solution:

$$-3x + 5 < -13$$
$$-3x < -13 - 5$$
$$-3x < -18$$
$$\frac{-3x}{-3} > \frac{-18}{-3}$$
$$x > 6$$

The solution set is $(6,\infty) = \{x \in \mathbb{R} | x > 6\}$.

15) The solution of the inequality $11 > 5 - 3x \geq -13$ is

Solution:

$$11 > 5 - 3x \geq -13$$
$$11 - 5 > -3x \geq -13 - 5$$
$$6 > -3x \geq -18$$
$$\frac{6}{-3} < \frac{-3x}{-3} \leq \frac{-18}{-3}$$
$$-2 < x \leq 6$$

The solution set is $(-2,6] = \{x \in \mathbb{R} | -2 < x \leq 6\}$.

16) If $2x + 3 = 1 - 6(x - 1)$, then $x =$

Solution:

$$2x + 3 = 1 - 6(x - 1)$$
$$2x + 3 = 1 - 6x + 6$$
$$2x + 6x = 1 + 6 - 3$$
$$8x = 4$$
$$x = \frac{4}{8}$$
$$x = \frac{1}{2}$$

17) The solution of the inequality $x^2 - 5x + 6 > 0$ is

Solution:

$$x^2 - 5x + 6 > 0$$
$$\Leftrightarrow \quad (x-2)(x-3) > 0$$

The transition points are 2 and 3. We should now investigate the sign of $(x-2)(x-3)$ where is > 0.

	2	3	
Sign of $(x-2)(x-3)$	$+++$	$---$	$+++$
Solution	Yes	No	Yes

The solution set is $(-\infty, 2) \cup (3, \infty)$.

18) The solution of the inequality $x^2 - 5x + 6 \geq 0$ is

Solution:

$$x^2 - 5x + 6 \geq 0$$
$$\Leftrightarrow \quad (x-2)(x-3) \geq 0$$

The transition points are 2 and 3. We should now investigate the sign of $(x-2)(x-3)$ where is ≥ 0.

	2	3	
Sign of $(x-2)(x-3)$	$+++$	$---$	$+++$
Solution	Yes	No	Yes

The solution set is $(-\infty, 2] \cup [3, \infty)$.

19) The solution of the inequality $x^2 - 5x + 6 \leq 0$ is

Solution:

$$x^2 - 5x + 6 \leq 0$$
$$\Leftrightarrow \quad (x-2)(x-3) \leq 0$$

The transition points are 2 and 3. We should now investigate the sign of $(x-2)(x-3)$ where is ≤ 0.

	2	3	
Sign of $(x-2)(x-3)$	$+++$	$---$	$+++$
Solution	No	Yes	No

The solution set is $[2,3]$.

20) The solution of the inequality $x^2 - 5x < -6$ is

Solution:

$$x^2 - 5x < -6$$
$$x^2 - 5x + 6 < 0$$
$$\Leftrightarrow \quad (x-2)(x-3) < 0$$

The transition points are 2 and 3. We should now investigate the sign of $(x-2)(x-3)$ where is < 0.

	2	3	
Sign of $(x-2)(x-3)$	$+++$	$---$	$+++$
Solution	No	Yes	No

The solution set is $(2,3)$.

21) The solution of the inequality $x^2 - x \geq 2$ is

Solution:

$$x^2 - x \geq 2$$
$$x^2 - x - 2 \geq 0$$
$$\Leftrightarrow \quad (x-2)(x+1) \geq 0$$

The transition points are -1 and 2. We should now investigate the sign of $(x-2)(x+1)$ where is ≥ 0.

	-1	2	
Sign of $(x-2)(x+1)$	$+++$	$---$	$+++$
Solution	Yes	No	Yes

The solution set is $(-\infty, -1] \cup [2, \infty)$.

22) The solution of the inequality $x^2 - x \leq 2$ is

Solution:

$$x^2 - x \leq 2$$
$$x^2 - x - 2 \leq 0$$
$$\Leftrightarrow \quad (x-2)(x+1) \leq 0$$

The transition points are -1 and 2. We should now investigate the sign of $(x-2)(x+1)$ where is ≤ 0.

	-1	2	
Sign of $(x-2)(x+1)$	$+++$	$---$	$+++$
Solution	No	Yes	No

The solution set is $[-1,2]$.

23) The solution of the inequality $x^2 - x > 2$ is

Solution:

$$x^2 - x > 2$$
$$x^2 - x - 2 > 0$$
$$\Leftrightarrow (x-2)(x+1) > 0$$

The transition points are -1 and 2. We should now investigate the sign of $(x-2)(x+1)$ where is > 0.

	-1		2
Sign of $(x-2)(x+1)$	$+++$	$---$	$+++$
Solution	Yes	No	Yes

The solution set is $(-\infty, -1) \cup (2, \infty)$.

24) If $|3x - 7| = 2$, then $x =$

Solution:

$$|3x - 7| = 2$$
$$3x - 7 = 2 \quad \text{or} \quad 3x - 7 = -2$$
$$3x = 2 + 7 \quad \text{or} \quad 3x = -2 + 7$$
$$3x = 9 \quad \text{or} \quad 3x = 5$$
$$x = \frac{9}{3} \quad \text{or} \quad x = \frac{5}{3}$$
$$x = 3 \quad \text{or} \quad x = \frac{5}{3}$$

25) If $|x - 4| = 3$, then $x =$

Solution:

$$|x - 4| = 3$$
$$x - 4 = 3 \quad \text{or} \quad x - 4 = -3$$
$$x = 3 + 4 \quad \text{or} \quad x = -3 + 4$$
$$x = 7 \quad \text{or} \quad x = 1$$

26) The solution of the inequality $|x - 3| < 4$ is

Solution:

$$|x - 3| < 4$$
$$-4 < x - 3 < 4$$
$$-4 + 3 < x < 4 + 3$$
$$-1 < x < 7$$

The solution set is $(-1, 7) = \{x \in \mathbb{R} | -1 < x < 7\}$.

27) The solution of the inequality $|x - 3| \leq 4$ is

Solution:

$$|x - 3| \leq 4$$
$$-4 \leq x - 3 \leq 4$$
$$-4 + 3 \leq x \leq 4 + 3$$
$$-1 \leq x \leq 7$$

The solution set is $[-1, 7] = \{x \in \mathbb{R} | -1 \leq x \leq 7\}$.

28) The solution of the inequality $|x - 3| > 4$ is

Solution:

$$|x - 3| > 4$$
$$x - 3 > 4 \quad or \quad x - 3 < -4$$
$$x > 4 + 3 \quad or \quad x < -4 + 3$$
$$x > 7 \quad or \quad x < -1$$

The solution set is $(-\infty, -1) \cup (7, \infty)$.

29) The solution of the inequality $|x - 3| \geq 4$ is

Solution:

$$|x - 3| \geq 4$$
$$x - 3 \geq 4 \quad or \quad x - 3 \leq -4$$
$$x \geq 4 + 3 \quad or \quad x \leq -4 + 3$$
$$x \geq 7 \quad or \quad x \leq -1$$

The solution set is $(-\infty, -1] \cup [7, \infty)$.

30) The distance between the real numbers -5 and 6 is

Solution:

$$\text{The distance } (d) = |(-5) - (6)| = |-11| = -(-11) = 11$$

31) The distance between the real numbers $\frac{15}{8}$ and $\frac{23}{12}$ is

Solution:

$$\text{The distance } (d) = \left|\left(\frac{15}{8}\right) - \left(\frac{23}{12}\right)\right| = \left|\frac{45 - 46}{24}\right| = \left|-\frac{1}{24}\right| = -\left(-\frac{1}{24}\right) = \frac{1}{24}$$

32) The distance between the points $(-2, -5)$ and $(3, 1)$ is

Solution:

$$d = \sqrt{(-2-3)^2 + (-5-1)^2} = \sqrt{(-5)^2 + (-6)^2} = \sqrt{25 + 36} = \sqrt{61}$$

33) The distance between the pairs $(-2, 5)$ and $(1, 1)$ is

Solution:

$$d = \sqrt{(-2-1)^2 + (5-1)^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

34) If $x^2 - 3x = 4$, then $x =$

Solution:

First, we write $x^2 - 3x - 4 = 0$

$$\Rightarrow (x-4)(x+1) = 0$$
$$\Rightarrow x - 4 = 0 \quad \text{or} \quad x + 1 = 0$$
$$\Rightarrow x = 4 \quad \text{or} \quad x = -1$$

35) If $3x^2 - 6 = 0$, then $x =$

Solution:

$$3x^2 - 6 = 0$$
$$\Rightarrow \quad 3x^2 = 6$$
$$\Rightarrow \quad x^2 = \frac{6}{3}$$
$$\Rightarrow \quad x^2 = 2$$
$$\Rightarrow \quad x = \pm\sqrt{2}$$

36) If $x(x-5) = 14$, then $x =$

Solution:

First, we write $x(x-5) = 14$

$$\Rightarrow \quad x^2 - 5x = 14$$
$$\Rightarrow \quad x^2 - 5x - 14 = 0$$
$$\Rightarrow \quad (x-7)(x+2) = 0$$
$$\Rightarrow \quad x - 7 = 0 \quad \text{or} \quad x + 2 = 0$$
$$\Rightarrow \quad x = 7 \quad \text{or} \quad x = -2$$

37) The solution of the equation

$$3(x-2) = 2(x+1) + 7 \quad \text{is}$$

Solution:

$$3(x-2) = 2(x+1) + 7$$
$$3x - 6 = 2x + 2 + 7$$
$$3x - 2x = 2 + 7 + 6$$
$$x = 15$$

38) The solution of the equation

$$2x + 3 = \frac{x}{2} + 9 \quad \text{is}$$

Solution:

$$2x + 3 = \frac{x}{2} + 9$$
$$4x + 6 = x + 18$$
$$4x - x = 18 - 6$$
$$3x = 12$$
$$x = 4$$

39) If $x^2 + 25 = 10x$, then $x =$

Solution:

$$x^2 + 25 = 10x$$
$$x^2 - 10x + 25 = 0$$
$$(x-5)(x-5) = 0$$
$$x = 5 \quad \text{(repeated)}$$

40) If $x^2 - 36 = 0$, then $x =$

Solution:

$$x^2 - 36 = 0$$
$$x^2 = 36$$
$$x = \pm\sqrt{36}$$
$$x = \pm 6$$

41) If $9(2x+8) = 20 - (x+5)$, then $x =$

Solution:

$$9(2x+8) = 20 - (x+5)$$
$$18x + 72 = 20 - x - 5$$
$$18x + x = 20 - 5 - 72$$
$$19x = -57$$
$$x = -\frac{57}{19}$$
$$x = -3$$

42) If $2(x-5) + 8 = 5x + 3$, then $x =$

Solution:

$$2(x-5) + 8 = 5x + 3$$
$$2x - 10 + 8 = 5x + 3$$
$$2x - 5x = 3 - 8 + 10$$
$$-3x = 5$$
$$x = -\frac{5}{3}$$

43) The solution of the equation

$$2x^2 - 3x = 5 \quad \text{is}$$

Solution:

$$2x^2 - 3x = 5$$
$$2x^2 - 3x - 5 = 0$$
$$a = 2, \; b = -3, \; c = -5$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{9 + 40}}{4} = \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4}$$
$$\therefore \quad x_1 = \frac{3+7}{4} = \frac{10}{4} = \frac{5}{2}$$
$$x_2 = \frac{3-7}{4} = \frac{-4}{4} = -1$$

Therefore, the solution of the equation is

$$x = -1 \quad \text{or} \quad x = \frac{5}{2}$$

44) The solution of the equation

$$x^3 - 2x^2 - 3x = 0 \quad \text{is}$$

Solution:

$$x^3 - 2x^2 - 3x = 0$$
$$x(x^2 - 2x - 3) = 0$$
$$x(x+1)(x-3) = 0$$
$$\Leftrightarrow \quad x = 0 \quad \text{or} \quad x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$\Leftrightarrow \quad x = 0 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 3$$

45) The solution of the equation

$$4x = \frac{2x+1}{3} - 2 \text{ is}$$

Solution:

$$4x = \frac{2x+1}{3} - 2$$
$$12x = (2x+1) - 6$$
$$12x = 2x + 1 - 6$$
$$12x - 2x = 1 - 6$$
$$10x = -5$$
$$x = \frac{-5}{10}$$
$$x = -\frac{1}{2}$$

46) The solution of the equation

$$x^4 + x^3 - 2x^2 = 0 \text{ is}$$

Solution:

$$x^4 + x^3 - 2x^2 = 0$$
$$x^2(x^2 + x - 2) = 0$$
$$x^2(x+2)(x-1) = 0$$
$$\Leftrightarrow \; x^2 = 0 \;\text{ or }\; x + 2 = 0 \;\text{ or }\; x - 1 = 0$$
$$\Leftrightarrow \; x = 0 \text{ (repeated) or } x = -2 \;\text{ or }\; x = 1$$

47) The solution of the equation

$$6x^2 + x = 2 \text{ is}$$

Solution:

$$6x^2 + x = 2$$
$$6x^2 + x - 2 = 0$$
$$a = 6,\; b = 1,\; c = -2$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1) \pm \sqrt{(1)^2 - 4(6)(-2)}}{2(6)}$$
$$= \frac{-1 \pm \sqrt{1 + 48}}{12} = \frac{-1 \pm \sqrt{49}}{12} = \frac{-1 \pm 7}{12}$$
$$\therefore \quad x_1 = \frac{-1+7}{12} = \frac{6}{12} = \frac{1}{2}$$
$$x_2 = \frac{-1-7}{12} = \frac{-8}{12} = -\frac{2}{3}$$

Therefore, the solution of the equation is

$$x = -\frac{2}{3} \text{ or } x = \frac{1}{2}$$

48) The solution of the equation

$$2x^2 + 3 = -7x \text{ is}$$

Solution:

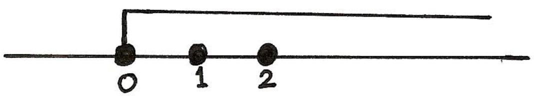
$$2x^2 + 3 = -7x$$
$$2x^2 + 7x + 3 = 0$$
$$a = 2,\; b = 7,\; c = 3$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-7 \pm \sqrt{49 - 24}}{4} = \frac{-7 \pm \sqrt{25}}{4}$$
$$= \frac{-7 \pm 5}{4}$$
$$\therefore \quad x_1 = \frac{-7+5}{4} = \frac{-2}{4} = -\frac{1}{2}$$
$$x_2 = \frac{-7-5}{4} = \frac{-12}{4} = -3$$

Therefore, the solution of the equation is

$$x = -\frac{1}{2} \text{ or } x = -3$$

49) $[0, \infty) \setminus \{1,2\} =$

Solution:



$$[0, \infty) \setminus \{1,2\} = [0,1) \cup (1,2) \cup (2, \infty).$$

50) The integer in $\mathbb{Z}$ is $\sqrt{25} = 5$.

51) The rational in $\mathbb{Q}$ is $\frac{2}{3}$.

52) The irrational in $\mathbb{I}$ is $\sqrt{2}$.