Workshop Solutions to Section 1.3

1) Find the slope of the line through the points $(-3,-6)$ and $(8,-5)$.
<u>Solution:</u>
The slope is
$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{(-6)-(-5)}{(-3)-(8)}=\frac{-6+5}{-3-8}=\frac{-1}{-11}=\frac{1}{11}$$

2) The slope of the line passes through $(2,6)$ and $(8,-3)$ is
<u>Solution:</u>
The slope is
$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{(6)-(-3)}{(2)-(8)}=\frac{6+3}{2-8}=\frac{9}{-6}=-\frac{3}{2}$$

3) The slope of the line passes through $(2,2)$ and $(-4,8)$ is
<u>Solution:</u>
The slope is
$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{(2)-(8)}{(2)-(-4)}=\frac{2-8}{2+4}=\frac{-6}{6}=-1$$

4) The slope of the line $2y=-6$ is
<u>Solution:</u>
First, we write the equation on the form
$$y=mx+c$$
So,
$$y=\frac{-6}{2}$$
$$y=-3$$
Thus, the slope $m=0$.

5) Find the equation of the line with slope -2 and y –intercept 3 is
<u>Solution:</u>
We apply the equation
$$y=mx+c$$
Thus,
$$y=-2x+3$$

6) Find the equation of the line through the point $(-3,4)$ with slope -2 .
<u>Solution:</u>
We apply the equation
$$y-y_1=m(x-x_1)$$
$$y-4=-2(x-(-3))$$
$$y-4=-2(x+3)$$
$$y-4=-2x-6$$
$$y=-2x-6+4$$
$$y=-2x-2$$

7) Find the equation of the line through the point $(1,2)$ with slope 5 .
<u>Solution:</u>
We apply the equation
$$y-y_1=m(x-x_1)$$
$$y-2=5(x-1)$$
$$y-2=5x-5$$
$$y=5x-5+2$$
$$y=5x-3$$

8) The equation of the line passes through the point $(-3,0)$ with slope 5 is
<u>Solution:</u>
We apply the equation
$$y-y_1=m(x-x_1)$$
$$y-0=5(x-(-3))$$
$$y=5(x+3)$$
$$y=5x+15$$

9) The equation of the line with slope $m=-2$ and passes through $(-5,1)$.
<u>Solution:</u>
We apply the equation
$$y-y_1=m(x-x_1)$$
$$y-1=-2(x-(-5))$$
$$y-1=-2(x+5)$$
$$y-1=-2x-10$$
$$y=-2x-10+1$$
$$y=-2x-9$$

10) Find the equation of the line through the points $(4,3)$ and $(2,8)$.

Solution:

First, we find the slope of the line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(3) - (8)}{(4) - (2)} = \frac{-5}{2} = -\frac{5}{2}$$

Then, we choose any one of the two points. Let us choose $(2,8)$ and apply the equation

$$y - y_1 = m(x - x_1)$$
$$y - 8 = -\frac{5}{2}(x - 2)$$
$$y - 8 = -\frac{5}{2}x + 5$$
$$y = -\frac{5}{2}x + 5 + 8$$
$$y = -\frac{5}{2}x + 13$$

11) The equation of the line passes through $(4,-3)$ and $(8,-5)$ is

Solution:

First, we find the slope of the line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(-3) - (-5)}{(4) - (8)} = \frac{-3 + 5}{4 - 8} = \frac{2}{-4} = -\frac{1}{2}$$

Then, we choose any one of the two points. Let us choose $(4,-3)$ and apply the equation

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -\frac{1}{2}(x - 4)$$
$$y + 3 = -\frac{1}{2}x + 2$$
$$y = -\frac{1}{2}x + 2 - 3$$
$$y = -\frac{1}{2}x - 1$$

or

$$y = -\frac{x}{2} - 1$$

12) The equation of the line passes through $(7,6)$ and $(8,9)$ is

Solution:

First, we find the slope of the line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(6) - (9)}{(7) - (8)} = \frac{-3}{-1} = 3$$

Then, we choose any one of the two points. Let us choose $(7,6)$ and apply the equation

$$y - y_1 = m(x - x_1)$$
$$y - 6 = 3(x - 7)$$
$$y - 6 = 3x - 21$$
$$y = 3x - 21 + 6$$
$$y = 3x - 15$$

13) The slope and the y –intercept of $2y - 3x = -6$ is

Solution:

We first should write the equation

$$2y - 3x = -6$$

on the form

$$y = mx + c$$

Thus,

$$2y - 3x = -6$$
$$2y = 3x - 6$$
$$y = \frac{3}{2}x - \frac{6}{2}$$
$$y = \frac{3}{2}x - 3$$

Therefore, the slope $m = \frac{3}{2}$ and

y –intercept $= -3$

14) Find the y –intercept of the line $3x - 2y - 1 = 0$.

Solution:

We first should write the equation

$$3x - 2y - 1 = 0$$

on the form

$$y = mx + c$$

Thus,

$$3x - 2y - 1 = 0$$
$$-2y = -3x + 1$$
$$y = \frac{-3}{-2}x + \frac{1}{-2}$$
$$y = \frac{3}{2}x - \frac{1}{2}$$

Therefore, the y –intercept $= -\frac{1}{2}$.

15) Find the slope of the perpendicular line to the line $5x - 2y - 1 = 0$.

Solution:

We first should write the equation

$$5x - 2y - 1 = 0$$

on the form

$$y = mx + c$$

Thus,

$$5x - 2y - 1 = 0$$
$$-2y = -5x + 1$$
$$y = \frac{-5}{-2}x + \frac{1}{-2}$$
$$y = \frac{5}{2}x - \frac{1}{2}$$

Therefore, the slope of the perpendicular line is $m = -\frac{2}{5}$.

16) Find the slope of the parallel line to the line $5x - 2y - 1 = 0$.

Solution:

We first should write the equation

$$5x - 2y - 1 = 0$$

on the form

$$y = mx + c$$

Thus,

$$5x - 2y - 1 = 0$$
$$-2y = -5x + 1$$
$$y = \frac{-5}{-2}x + \frac{1}{-2}$$
$$y = \frac{5}{2}x - \frac{1}{2}$$

Therefore, the slope of the parallel line is $m = \frac{5}{2}$.

17) The slope of the perpendicular line to the line $3y + 2x - 6 = 0$ is

Solution:

We first should write the equation

$$3y + 2x - 6 = 0$$

on the form

$$y = mx + c$$

Thus,

$$3y + 2x - 6 = 0$$
$$3y = -2x + 6$$
$$y = \frac{-2}{3}x + \frac{6}{3}$$
$$y = -\frac{2}{3}x + 2$$

Therefore, the slope of the perpendicular line is $m = \frac{3}{2}$.

18) The slope of the parallel line to the line $3y + 2x - 6 = 0$ is

Solution:

We first should write the equation

$$3y + 2x - 6 = 0$$

on the form

$$y = mx + c$$

Thus,

$$3y + 2x - 6 = 0$$
$$3y = -2x + 6$$
$$y = \frac{-2}{3}x + \frac{6}{3}$$
$$y = -\frac{2}{3}x + 2$$

Therefore, the slope of the parallel line is $m = -\frac{2}{3}$.

19) The equation for the line passes through $(-2, -1)$ and parallel to the line $2x + 5y - 10 = 0$ is

Solution:

First, we have to find the slope of the line. Since the two lines are parallel, then they have the same slope. Now, we write the equation

$$2x + 5y - 10 = 0$$

on the form

$$y = mx + c$$

Thus,

$$2x + 5y - 10 = 0$$
$$5y = -2x + 10$$
$$y = \frac{-2}{5}x + \frac{10}{5}$$
$$y = -\frac{2}{5}x + 2$$

Therefore, the slope is $m = -\frac{2}{5}$ and the required line is

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{5}(x - (-2))$$
$$y + 1 = -\frac{2}{5}(x + 2)$$
$$y + 1 = -\frac{2}{5}x - \frac{4}{5}$$
$$y = -\frac{2}{5}x - \frac{4}{5} - 1$$
$$y = -\frac{2}{5}x - \frac{9}{5}$$

20) The equation for the line passes through $(4,-1)$ and parallel to the line $2x-y=3$ is

Solution:

First, we have to find the slope of the line. Since the two lines are parallel, then they have the same slope. Now, we write the equation

$$2x-y=3$$

on the form

$$y=mx+c$$

Thus,

$$2x-y=3$$
$$-y=-2x+3$$
$$y=\frac{-2}{-1}x+\frac{3}{-1}$$
$$y=2x-3$$

Therefore, the slope is $m=2$ and the required line is

$$y-y_1=m(x-x_1)$$
$$y-(-1)=2(x-4)$$
$$y+1=2x-8$$
$$y=2x-8-1$$
$$y=2x-9$$

OR

$$y-2x=-9$$

21) The equation for the line passes through $(1,4)$ and parallel to the line $2x-6y+5=0$ is

Solution:

First, we have to find the slope of the line. Since the two lines are parallel, then they have the same slope. Now, we write the equation

$$2x-6y+5=0$$

on the form

$$y=mx+c$$

Thus,

$$2x-6y+5=0$$
$$-6y=-2x-5$$
$$y=\frac{-2}{-6}x-\frac{5}{-6}$$
$$y=\frac{1}{3}x+\frac{5}{6}$$

Therefore, the slope is $m=\frac{1}{3}$ and the required line is

$$y-y_1=m(x-x_1)$$
$$y-4=\frac{1}{3}(x-1)$$
$$3y-12=x-1$$
$$3y=x-1+12$$
$$3y=x+11$$

22) The equation for the line passes through $(-3,6)$ and perpendicular to the line $3x-y-8=0$ is

Solution:

First, we have to find the slope of the line. Since the two lines are perpendicular, then the product of their slopes equals to -1. Now, we write the equation $3x-y-8=0$ on the form $y=mx+c$

Thus,

$$3x-y-8=0$$
$$-y=-3x+8$$
$$y=\frac{-3}{-1}x+\frac{8}{-1}$$
$$y=3x-8$$

Therefore, the slope is $m=-\frac{1}{3}$ and the required line is

$$y-y_1=m(x-x_1)$$
$$y-6=-\frac{1}{3}(x-(-3))$$
$$y-6=-\frac{1}{3}(x+3)$$
$$y-6=-\frac{1}{3}x-1$$
$$y=-\frac{1}{3}x-1+6$$
$$y=-\frac{1}{3}x+5$$

23) The equation for the line passes through $(4,-1)$ and perpendicular to the line $2x-y=3$ is

Solution:

First, we have to find the slope of the line. Since the two lines are perpendicular, then the product of their slopes equals to -1. Now, we write the equation $2x-y=3$ on the form $y=mx+c$

Thus,

$$2x-y=3$$
$$-y=-2x+3$$
$$y=\frac{-2}{-1}x+\frac{3}{-1}$$
$$y=2x-3$$

Therefore, the slope is $m=-\frac{1}{2}$ and the required line is

$$y-y_1=m(x-x_1)$$
$$y-(-1)=-\frac{1}{2}(x-4)$$
$$y+1=-\frac{1}{2}(x-4)$$
$$2y+2=-(x-4)$$
$$2y+2=-x+4$$
$$2y+x=4-2$$
$$2y+x=2$$

24) The equation for the line passes through $(1,4)$ and perpendicular to the line $2x - 6y + 5 = 0$ is

<u>Solution:</u>

First, we have to find the slope of the line. Since the two lines are perpendicular, then the product of their slopes equals to -1. Now, we write the equation $2x - 6y + 5 = 0$ on the form $y = mx + c$

Thus,

$$2x - 6y + 5 = 0$$
$$-6y = -2x - 5$$
$$y = \frac{-2}{-6}x - \frac{5}{-6}$$
$$y = \tfrac{1}{3}x + \tfrac{5}{6}$$

Therefore, the slope is $m = -3$ and the required line is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -3(x - 1)$$
$$y - 4 = -3x + 3$$
$$3x + y = 3 + 4$$
$$3x + y = 7$$

25) The slope of the line $2x = -6$ is

<u>Solution:</u>

It is an undefined.

26) The equation of the vertical line passes through $(-3,-6)$ is

<u>Solution:</u>

$$x = -3$$

27) The equation of the horizontal line passes through $(-3,-6)$ is

<u>Solution:</u>

$$y = -6$$

28) The equation of the line with slope $m = \frac{2}{9}$ and y −intercept 4 is

<u>Solution:</u>

We apply the equation

$$y = mx + c$$

Thus,

$$y = \frac{2}{9}x + 4$$

29) The equation of the line with slope $m = -3$ and passes through the point of the intersection of the two lines $3x - y + 1 = 0$ and $y = 2x + 3$ is

<u>Solution:</u>

We first have to find the point of the intersection of the two lines.

$3x - y + 1 = 0$ and $y = 2x + 3$ ------(1)

$-y = -3x - 1$

$y = 3x + 1$ ------(2)

Now, make (1)=(2)

$$2x + 3 = 3x + 1$$
$$2x - 3x = 1 - 3$$
$$-x = -2$$
$$x = 2$$

Substitute into (1) to find y, so

$$y = 2(2) + 3 = 4 + 3 = 7$$

Thus, the point is $(x, y) = (2,7)$.

Therefore, the required line is

$$y - y_1 = m(x - x_1)$$
$$y - 7 = -3(x - 2)$$
$$y - 7 = -3x + 6$$
$$y = -3x + 6 + 7$$
$$y = -3x + 13$$

30) The midpoint of the segment with endpoints $(4,-9)$ and $(-12,-3)$ is

<u>Solution:</u>

The midpoint

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + (-12)}{2}, \frac{(-9) + (-3)}{2}\right)$$
$$= \left(\frac{4 - 12}{2}, \frac{-9 - 3}{2}\right) = \left(\frac{-8}{2}, \frac{-12}{2}\right)$$
$$= (-4,-6)$$

31) The midpoint of the segment with endpoints $(\sqrt{3},-1)$ and $(3\sqrt{3},4)$ is

<u>Solution:</u>

The midpoint

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{(\sqrt{3}) + (3\sqrt{3})}{2}, \frac{(-1) + (4)}{2}\right)$$
$$= \left(\frac{\sqrt{3} + 3\sqrt{3}}{2}, \frac{-1 + 4}{2}\right) = \left(\frac{4\sqrt{3}}{2}, \frac{3}{2}\right)$$
$$= \left(2\sqrt{3}, \frac{3}{2}\right)$$

32) The midpoint of the segment with endpoints $(-3,-1)$ and $(9,4)$ is

Solution:

The midpoint

$$=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{(-3)+(9)}{2},\frac{(-1)+(4)}{2}\right)$$

$$=\left(\frac{-3+9}{2},\frac{-1+4}{2}\right)=\left(\frac{6}{2},\frac{3}{2}\right)=\left(3,\frac{3}{2}\right)$$

33) The intersection point of the lines $y=-2$ and $x=3$ is

Solution:

It is $(x,y)=(3,-2)$.

34) The equation for the line passes through $\left(\frac{1}{2},-\frac{2}{3}\right)$ and parallel to the line $4x-8y-1=0$ is

Solution:

First, we have to find the slope of the line. Since the two lines are parallel, then they have the same slope. Now, we write the equation

$$4x-8y-1=0$$

on the form

$$y=mx+c$$

Thus,

$$4x-8y-1=0$$
$$-8y=-4x+1$$
$$y=\frac{-4}{-8}x+\frac{1}{-8}$$
$$y=\frac{1}{2}x-\frac{1}{8}$$

Therefore, the slope is $m=\frac{1}{2}$ and the required line is

$$y-y_1=m(x-x_1)$$
$$y-\left(-\frac{2}{3}\right)=\frac{1}{2}\left(x-\frac{1}{2}\right)$$
$$y+\frac{2}{3}=\frac{1}{2}x-\frac{1}{4}$$
$$y=\frac{1}{2}x-\frac{1}{4}-\frac{2}{3}$$
$$y=\frac{1}{2}x-\frac{11}{12}$$

35) The equation for the line passes through $\left(\frac{1}{2},-\frac{2}{3}\right)$ and perpendicular to the line $4x-8y-1=0$ is

Solution:

First, we have to find the slope of the line. Since the two lines are perpendicular, then the product of their slopes equals to -1 . Now, we write the equation $4x-8y-1=0$ on the form $y=mx+c$

Thus,

$$4x-8y-1=0$$
$$-8y=-4x+1$$
$$y=\frac{-4}{-8}x+\frac{1}{-8}$$
$$y=\frac{1}{2}x-\frac{1}{8}$$

Therefore, the slope is $m=-2$ and the required line is

$$y-y_1=m(x-x_1)$$
$$y-\left(-\frac{2}{3}\right)=-2\left(x-\frac{1}{2}\right)$$
$$y+\frac{2}{3}=-2x+1$$
$$y=-2x+1-\frac{2}{3}$$
$$y=-2x+\frac{1}{3}$$

36) Find the equation of the line through $(6\sqrt{2},-\sqrt{2})$ with slope $-\frac{1}{2}$.

Solution:

We apply the equation

$$y-y_1=m(x-x_1)$$
$$y-\left(-\sqrt{2}\right)=-\frac{1}{2}\left(x-6\sqrt{2}\right)$$
$$y+\sqrt{2}=-\frac{1}{2}x+\frac{6\sqrt{2}}{2}$$
$$y+\sqrt{2}=-\frac{1}{2}x+3\sqrt{2}$$
$$y=-\frac{1}{2}x+3\sqrt{2}-\sqrt{2}$$
$$y=-\frac{1}{2}x+2\sqrt{2}$$

37) Find the equation of the line through $\left(6\sqrt{2},-\sqrt{2}\right)$ and parallel to the line with slope $-\frac{1}{2}$

Solution:

Since the two lines are parallel, then they have the same slope. Now, we apply the equation

$$y-y_1=m(x-x_1)$$
$$y-\left(-\sqrt{2}\right)=-\frac{1}{2}\left(x-6\sqrt{2}\right)$$
$$y+\sqrt{2}=-\frac{1}{2}x+\frac{6\sqrt{2}}{2}$$
$$y+\sqrt{2}=-\frac{1}{2}x+3\sqrt{2}$$
$$y=-\frac{1}{2}x+3\sqrt{2}-\sqrt{2}$$
$$y=-\frac{1}{2}x+2\sqrt{2}$$

38) The equation of the line segment joining the points $(1,4)$ and $(7,-2)$ is

Solution:

First, we find the slope of the line segment

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{(4)-(-2)}{(1)-(7)}=\frac{4+2}{1-7}=\frac{6}{-6}=-1$$

Then, we choose any one of the two points. Let us choose $(7,-2)$ and apply the equation

$$y-y_1=m(x-x_1)$$
$$y-(-2)=(-1)(x-7)$$
$$y+2=-x+7$$
$$y=-x+7-2$$
$$y=-x+5$$

39) Find the equation for the line passes through the point $\left(\frac{1}{2},-\frac{2}{3}\right)$ and perpendicular to the line segment joining the points $(1,4)$ and $(7,-2)$.

Solution:

First, we find the slope of the perpendicular line

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{(4)-(-2)}{(1)-(7)}=\frac{4+2}{1-7}=\frac{6}{-6}=-1$$

Since the two lines are perpendicular, then the product of their slopes equals to -1. Hence, the slope of the line is 1. Now, we apply the equation

$$y-y_1=m(x-x_1)$$
$$y-\left(-\frac{2}{3}\right)=(1)\left(x-\frac{1}{2}\right)$$
$$y+\frac{2}{3}=x-\frac{1}{2}$$
$$y=x-\frac{1}{2}-\frac{2}{3}$$
$$y=x-\frac{7}{6}$$

40) Find the equation for the line passes through the point $\left(\frac{1}{2},-\frac{2}{3}\right)$ and parallel to the line segment joining the points $(1,4)$ and $(7,-2)$.

Solution:

First, we find the slope of the parallel line

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{(4)-(-2)}{(1)-(7)}=\frac{4+2}{1-7}=\frac{6}{-6}=-1$$

Since the two lines are parallel, then they have the same slope. Now, we apply the equation

$$y-y_1=m(x-x_1)$$
$$y-\left(-\frac{2}{3}\right)=(-1)\left(x-\frac{1}{2}\right)$$
$$y+\frac{2}{3}=-x+\frac{1}{2}$$
$$y=-x+\frac{1}{2}-\frac{2}{3}$$
$$y=-x-\frac{1}{6}$$