Workshop Solutions to Section 1.3

1) Find the slope of the line through the points (-3, -6) and (8, -5). <u>Solution:</u> The slope is $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(-6) - (-5)}{(-3) - (8)} = \frac{-6 + 5}{-3 - 8} = \frac{-1}{-11} = \frac{1}{11}$	2) The slope of the line passes through (2,6) and (8,-3) is <u>Solution:</u> The slope is $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(6) - (-3)}{(2) - (8)} = \frac{6+3}{2-8} = \frac{9}{-6} = -\frac{3}{2}$
3) The slope of the line passes through (2,2) and (-4,8) is <u>Solution:</u> The slope is $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(2) - (8)}{(2) - (-4)} = \frac{2 - 8}{2 + 4} = \frac{-6}{6} = -1$	4) The slope of the line $2y = -6$ is <u>Solution:</u> First, we write the equation on the form y = mx + c So, $y = \frac{-6}{2}$ y = -3 Thus, the slope $m = 0$.
5) Find the equation of the line with slope -2 and y -intercept 3 is Solution: We apply the equation y = mx + c Thus, y = -2x + 3	6) Find the equation of the line through the point (-3,4) with slope -2. Solution: We apply the equation $y - y_1 = m(x - x_1)$ $y - 4 = -2(x - (-3))$ $y - 4 = -2(x + 3)$ $y - 4 = -2x - 6$ $y = -2x - 6 + 4$ $y = -2x - 2$
7) Find the equation of the line through the point (1,2) with slope 5. Solution: We apply the equation $y - y_1 = m(x - x_1)$ y - 2 = 5(x - 1) y - 2 = 5x - 5 y = 5x - 5 + 2 y = 5x - 3	8) The equation of the line passes through the point (-3,0) with slope 5 is <u>Solution:</u> We apply the equation $y - y_1 = m(x - x_1)$ y - 0 = 5(x - (-3)) y = 5(x + 3) y = 5x + 15
9) The equation of the line with slope $m = -2$ and passes through $(-5,1)$. Solution: We apply the equation $y - y_1 = m(x - x_1)$ y - 1 = -2(x - (-5)) y - 1 = -2(x + 5) y - 1 = -2x - 10 y = -2x - 10 + 1 y = -2x - 9	

10) Find the equation of the line through the	11) The equation of the line passes through
points (4,3) and (2,8) .	(4,-3) and (8,-5) is
Solution:	Solution:
First, we find the slope of the line	First, we find the slope of the line (2)
$m = \frac{y_1 - y_2}{(4)} = \frac{(3) - (8)}{(4)} = \frac{-5}{2} = -\frac{5}{2}$	$m = \frac{y_1 - y_2}{x_1 - y_2} = \frac{(-3) - (-3)}{(4)} = \frac{-3 + 5}{4} = \frac{2}{4} = -\frac{1}{2}$
$x_1 - x_2$ (4) - (2) 2 2 Then we choose any one of the two points let	$x_1 - x_2$ (4) - (8) 4 - 8 - 4 2
Inen, we choose any one of the two points. Let	Inen, we choose any one of the two points. Let
us choose (2,8) and apply the equation $y = y = m(r - r_{c})$	us choose $(4, -3)$ and apply the equation $y - y = m(r - r)$
$\begin{array}{c} y y_1 = m(x x_1) \\ 5 \\ \end{array}$	$\begin{array}{c} y y_1 = m(x x_1) \\ 1 \\ \end{array}$
$y - 8 = -\frac{1}{2}(x - 2)$	$y - (-3) = -\frac{1}{2}(x - 4)$
$y = 8 = -\frac{5}{2}x + 5$	$y + 3 = -\frac{1}{2}r + 2$
$y - 8 = -\frac{1}{2}x + 3$	$y + 3 = -\frac{2}{2}x + 2$
$y = -\frac{5}{2}x + 5 + 8$	$y = -\frac{1}{2}x + 2 - 3$
$y = -\frac{5}{2}x + 13$	$y = -\frac{1}{2}x - 1$
Ζ	or
	$y = -\frac{x}{x} - 1$
12) The equation of the line response through	$\frac{2}{2}$
(7.6) and (8.0) is	13) The slope and the y –intercept of $2y - 2x = -6$ is
	2y - 5x = -0 is
<u>Solution.</u> First we find the slope of the line	We first should write the equation
$v_1 - v_2$ (6) - (9) -3	2y - 3x = -6
$m = \frac{y_1}{y_1 - y_2} = \frac{(3)}{(7) - (8)} = \frac{3}{-1} = 3$	on the form
Then, we choose any one of the two points. Let	y = mx + c
us choose (7,6) and apply the equation	Thus,
$y - y_1 = m(x - x_1)$	2y - 3x = -6
y - 6 = 3(x - 7)	2y = 3x - 6
y-6=3x-21	$y = \frac{3}{2}x - \frac{6}{2}$
y = 3x - 21 + 6	2 2 3
y = 3x - 15	$y = \frac{1}{2}x - 3$
	Therefore, the slope $m = \frac{3}{2}$ and
	v - intercept = -3
14) Find the y –intercept of the line	15) Find the slope of the perpendicular line to
3x - 2y - 1 = 0	the line $5x - 2y - 1 = 0$.
Solution:	Solution:
We first should write the equation	We first should write the equation
3x - 2y - 1 = 0	5x - 2y - 1 = 0
on the form	on the form
y = mx + c	y = mx + c
2x - 2y - 1 = 0	5x - 2y - 1 = 0
3x - 2y - 1 = 0 -2y = -3x + 1	-2y = -5x + 1
-3 1	-5 1
$y = \frac{1}{-2}x + \frac{1}{-2}$	$y = \frac{1}{-2}x + \frac{1}{-2}$
$y = \frac{3}{r} - \frac{1}{r}$	$y = \frac{5}{5}x = \frac{1}{2}$
$y = 2^{x} - 2^{x}$	$y - \frac{1}{2}x - \frac{1}{2}$
Therefore, the y -intercept = $-\frac{1}{2}$.	Inerefore, the slope of the perpendicular line is $\frac{1}{2}$
	$m = -\frac{2}{5}$.

16) Find the slope of the parallel line to the	17) The slope of the perpendicular line to the
line $5x - 2y - 1 = 0$.	line $3y + 2x - 6 = 0$ is
Solution:	Solution:
We first should write the equation	We first should write the equation
5x - 2y - 1 = 0	3y + 2x - 6 = 0
on the form	on the form
y = mx + c	y = mx + c
Thus,	Thus,
5x - 2y - 1 = 0	3y + 2x - 6 = 0
-2y = -5x + 1	3y = -2x + 6
$y = \frac{-5}{-2}x + \frac{1}{-2}$	$y = \frac{-2}{2}x + \frac{6}{2}$
-2 -2 5 1	2 3 3
$y = \frac{3}{2}x - \frac{1}{2}$	$y = -\frac{2}{3}x + 2$
Therefore, the slope of the parallel line is	Therefore, the slope of the perpendicular line is
$m = \frac{5}{2}$	$m = \frac{3}{2}$
$m = \frac{1}{2}$.	$m = \frac{1}{2}$
18) The slope of the parallel line to the line	19) The equation for the line passes through
3y + 2x - 6 = 0 is	(-2, -1) and parallel to the line $2x + 5y - 10 = 0$
Solution:	IS
We first should write the equation	Solution:
3y + 2x - 6 = 0	First, we have to find the slope of the line.
on the form	Since the two lines are parallel, then they have
y = mx + c	the same slope. Now, we write the equation
2n + 2r = 6 = 0	2x + 5y - 10 = 0
3y + 2x = 0 = 0 $3y = -2x \pm 6$	
-2 = -2 = 6	y = mx + c
$y = \frac{1}{3}x + \frac{3}{3}$	$2x \pm 5y = 10 = 0$
2	5y = -2x + 10
$y = -\frac{1}{3}x + 2$	-2 10
Therefore, the slope of the parallel line is	$y = \frac{1}{5}x + \frac{1}{5}$
$m = -\frac{2}{2}$.	2
3	$y = -\frac{1}{5}x + 2$
	Therefore, the slope is $m = -\frac{2}{2}$ and the
	required line is
	$v - v_1 = m(x - x_1)$
	$y - (-1) = -\frac{1}{5}(x - (-2))$
	2 + 1 - 2(x + 2)
	$y + 1 = -\frac{1}{5}(x + 2)$
	$y + 1 = -\frac{2}{r} - \frac{4}{r}$
	5, 5
	$y = -\frac{2}{\pi}x - \frac{4}{\pi} - 1$
	2 9
	$y = -\frac{z}{5}x - \frac{z}{5}$
	5 5

20) The equation for the line passes through	21) The equation for the line passes through
(4, -1) and parallel to the line $2x - y = 3$ is	(1,4) and parallel to the line $2x - 6y + 5 = 0$ is
Solution:	Solution:
First, we have to find the slope of the line.	First, we have to find the slope of the line.
Since the two lines are parallel, then they have	Since the two lines are parallel, then they have
the same slope. Now, we write the equation	the same slope. Now, we write the equation
2x - y = 3	2x - 6y + 5 = 0
on the form	on the form
y = mx + c	y = mx + c
2x - y = 2	2x - 6y + 5 = 0
2x - y = 3 $-y = -2r + 3$	2x - 0y + 3 = 0 -6y = -2x = 5
y = -2x + 3 -2 3	-2 5
$y = \frac{1}{-1}x + \frac{1}{-1}$	$y = \frac{1}{-6}x - \frac{1}{-6}$
y = 2x - 3	1 5
Therefore, the slope is $m = 2$ and the required	$y = \frac{1}{3}x + \frac{1}{6}$
line is	Therefore, the slope is $m = \frac{1}{2}$ and the required
$y - y_1 = m(x - x_1)$	line is
y - (-1) = 2(x - 4)	$y - y_1 = m(x - x_1)$
y + 1 = 2x - 8	
y = 2x - 8 - 1	$y - 4 = \frac{1}{3}(x - 1)$
y = 2x - 9	3y - 12 = x - 1
OR	3y = x - 1 + 12
y - 2x = -9	3y = x + 11
22) The equation for the line passes through	23) The equation for the line passes through
(-3,6) and perpendicular to the line	(4, -1) and perpendicular to the line
3x - y - 8 = 0 is	2x - y = 3 is
Solution:	Solution:
First, we have to find the slope of the line.	First, we have to find the slope of the line.
Since the two lines are perpendicular, then the	Since the two lines are perpendicular, then the
product of their slopes equals to -1 . Now, we	product of their slopes equals to -1 . Now, we
write the equation $3x - y - 8 = 0$	write the equation $2x - y = 3$
on the form $y = mx + c$	on the form $y = mx + c$
2x y 8 = 0	2x y = 2
5x - y - 6 = 0 -y = -3r + 8	2x - y - 3 $-y - 2r + 3$
y = -3 8	y = 2x + 3 -2 3
$y = \frac{1}{-1}x + \frac{1}{-1}$	$y = \frac{1}{-1}x + \frac{1}{-1}$
y = 3x - 8	$y = 2x - 3^{-1}$
Therefore, the slope is $m = -\frac{1}{2}$ and the	Therefore, the slope is $m = -\frac{1}{2}$ and the
required line is	required line is
$v - v_1 = m(x - x_1)$	$v - v_1 = m(x - x_1)$
$y - 6 = -\frac{1}{3}(x - (-3))$	$y - (-1) = -\frac{1}{2}(x - 4)$
$y - 6 = -\frac{1}{-1}(x + 3)$	$y + 1 = -\frac{1}{2}(x - 4)$
3 (** + 5) 1	$2^{(x-y)}$ 2y + 2 = -(x - 4)
$y - 6 = -\frac{1}{3}x - 1$	2y + 2 = -(x - 4) $2y + 2 = -x + 4$
1	2v + x = 4 - 2
$y = -\frac{1}{3}x - 1 + 6$	2y + x = 2
$y = -\frac{1}{3}x + 5$	

24) The equation for the line passes through (1,4) and perpendicular to the line $2x - 6y + 5 = 0$ is	25) The slope of the line $2x = -6$ is <u>Solution</u> : It is an undefined.
Solution: First, we have to find the slope of the line. Since the two lines are perpendicular, then the product of their slopes equals to -1 . Now, we write the equation $2x - 6y + 5 = 0$	26) The equation of the vertical line passes through $(-3, -6)$ is <u>Solution:</u> $x = -3$
on the form $y = mx + c$ Thus, 2x - 6y + 5 = 0 -6y = -2x - 5 $y = \frac{-2}{-6}x - \frac{5}{-6}$	27) The equation of the horizontal line passes through $(-3, -6)$ is <u>Solution:</u> $y = -6$
$y = \frac{1}{3}x + \frac{5}{6}$ Therefore, the slope is $m = -3$ and the required line is $y - y_1 = m(x - x_1)$ $y - 4 = -3(x - 1)$ $y - 4 = -3x + 3$ $3x + y = 3 + 4$ $3x + y = 7$	28) The equation of the line with slope $m = \frac{2}{9}$ and <i>y</i> -intercept 4 is <u>Solution:</u> We apply the equation y = mx + c Thus, $y = \frac{2}{9}x + 4$
29) The equation of the line with slope $m = -3$ and passes through the point of the intersection of the two lines $3x - y + 1 = 0$ and y = 2x + 3 is Solution: We first have to find the point of the intersection of the two lines. 3x - y + 1 = 0 and $y = 2x + 3$ (1) -y = -3x - 1 y = 3x + 1(2)	30) The midpoint of the segment with endpoints (4, -9) and (-12, -3) is <u>Solution:</u> The midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + (-12)}{2}, \frac{(-9) + (-3)}{2}\right)$ $= \left(\frac{4 - 12}{2}, \frac{-9 - 3}{2}\right) = \left(\frac{-8}{2}, \frac{-12}{2}\right)$ $= (-4, -6)$
Now, make (1)=(2) 2x + 3 = 3x + 1 2x - 3x = 1 - 3 -x = -2 x = 2 Substitute into (1) to find y, so y = 2(2) + 3 = 4 + 3 = 7 Thus, the point is $(x, y) = (2,7)$. Therefore, the required line is $y - y_1 = m(x - x_1)$ y - 7 = -3(x - 2) y - 7 = -3x + 6 y = -3x + 6 + 7 y = -3x + 13	31) The midpoint of the segment with endpoints $(\sqrt{3}, -1)$ and $(3\sqrt{3}, 4)$ is <u>Solution:</u> The midpoint $=\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $=\left(\frac{(\sqrt{3}) + (3\sqrt{3})}{2}, \frac{(-1) + (4)}{2}\right)$ $=\left(\frac{\sqrt{3} + 3\sqrt{3}}{2}, \frac{-1 + 4}{2}\right) = \left(\frac{4\sqrt{3}}{2}, \frac{3}{2}\right)$ $=\left(2\sqrt{3}, \frac{3}{2}\right)$

32) The midpoint of the segment with	33) The intersection point of the lines $y = -2$
endpoints $(-3, -1)$ and $(9, 4)$ is	and $x = 3$ is
Solution:	Solution:
The midpoint	It is $(x, y) = (3, -2)$.
$=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{(-3)+(9)}{2}, \frac{(-1)+(4)}{2}\right)$	
(2 2 7 (2 2 7))	
$=\left(\frac{3+7}{2},\frac{1+1}{2}\right)=\left(\frac{3}{2},\frac{3}{2}\right)=\left(3,\frac{3}{2}\right)$	
24) The equation for the line process through	2E) The equation for the line passes through
$\binom{1}{2}$ The equation for the line passes through	$\binom{35}{1}$ The equation for the line passes through $\binom{1}{2}$
$\left(\frac{-}{2},-\frac{-}{3}\right)$ and parallel to the line $4x-8y-1=0$ is	$\left(\frac{1}{2},-\frac{1}{3}\right)$ and perpendicular to the line
Solution:	4x - 8y - 1 = 0 is
First, we have to find the slope of the line.	Solution:
Since the two lines are parallel, then they have	First, we have to find the slope of the line.
the same slope. Now, we write the equation	Since the two lines are perpendicular, then the
4x - 8y - 1 = 0	product of their slopes equals to -1 . Now, we
on the form	write the equation $4x - 8y - 1 = 0$
y = mx + c	on the form $y = mx + c$
Thus,	Thus,
4x - 8y - 1 = 0	4x - 8y - 1 = 0
-8y = -4x + 1	-8y = -4x + 1
$y = \frac{-4}{2}x + \frac{1}{2}$	$y = \frac{-4}{-2}x + \frac{1}{-2}$
$ \begin{array}{ccc} -8 & -8 \\ 1 & 1 \end{array} $	
$y = \frac{1}{2}x - \frac{1}{8}$	$y = \frac{1}{2}x - \frac{1}{8}$
Therefore, the slope is $m = \frac{1}{2}$ and the required	Therefore, the slope is $m = -2$ and the
	required line is
$y - y_t = m(r - r_t)$	$y - y_1 = m(x - x_1)$
$y y_1 - m(x x_1)$ (2) 1(1)	$(\frac{2}{2}) = 2(x^{-1})$
$y - (-\frac{1}{3}) = \frac{1}{2}(x - \frac{1}{2})$	$y - (-\frac{1}{3}) = -2(x - \frac{1}{2})$
	$v + \frac{2}{-2} = -2x + 1$
$y + \frac{1}{3} = \frac{1}{2}x - \frac{1}{4}$	3 20 1 2
$y = \frac{1}{x} - \frac{1}{x} - \frac{2}{x}$	$y = -2x + 1 - \frac{2}{2}$
2^{1}_{1} 4^{1}_{11} 3^{1}_{11}	1 3
$y = \frac{1}{2}x - \frac{11}{12}$	$y = -2x + \frac{1}{3}$
$\frac{2}{36}$ Eind the equation of the line through	37) Find the equation of the line through
$(6\sqrt{2} - \sqrt{2})$ with slope $-\frac{1}{2}$	$(6\sqrt{2} - \sqrt{2})$ and parallel to the line with slope $-\frac{1}{2}$
$\sum_{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n$	$(0,2), (0,2)$ and parameter to the line with slope $\frac{1}{2}$
Solution.	Since the two lines are parallel, then they have
we apply the equation $y = y = m(x - x)$	the same slope. New, we apply the equation
$y - y_1 - m(x - x_1)$	the same slope. Now, we apply the equation $y - y_{1} = m(r - r_{1})$
$y - (-\sqrt{2}) = -\frac{1}{2}(x - 6\sqrt{2})$	$y y_1 = m(x x_1)$
$-\frac{1}{1}$ $6\sqrt{2}$	$y - (-\sqrt{2}) = -\frac{1}{2}(x - 6\sqrt{2})$
$y + \sqrt{2} = -\frac{1}{2}x + \frac{1}{2}$	$-\frac{1}{6\sqrt{2}}$
$\frac{1}{1}$ $\frac{1}$	$y + \sqrt{2} = -\frac{1}{2}x + \frac{1}{2}$
$y + v 2 - \frac{1}{2}x + 3v 2$	$y + \sqrt{2} - \frac{1}{r} + \frac{3}{2}$
$y = -\frac{1}{2}x + 3\sqrt{2} - \sqrt{2}$	$y + v^2 - 2^{x + 3v^2}$
	$y = -\frac{1}{2}x + 3\sqrt{2} - \sqrt{2}$
$y = -\frac{1}{2}x + 2\sqrt{2}$	
2	$y = -\frac{1}{2}x + 2\sqrt{2}$

38) The equation of the line segment joining the points (1,4) and (7,-2) is <u>Solution:</u> First, we find the slope of the line segment $y_1 - y_2$ (4) - (-2) 4 + 2 6	39) Find the equation for the line passes through the point $(\frac{1}{2}, -\frac{2}{3})$ and perpendicular to the line segment joining the points (1,4) and (7,-2).
$m = \frac{y_1 - x_2}{x_1 - x_2} = \frac{(y_1 - (x_1))}{(1) - (7)} = \frac{1}{1 - 7} = \frac{1}{-6} = -1$ Then, we choose any one of the two points. Let us choose (7, -2) and apply the equation $y - y_1 = m(x - x_1)$ $y - (-2) = (-1)(x - 7)$ $y + 2 = -x + 7$ $y = -x + 7 - 2$ $y = -x + 5$	Solution: First, we find the slope of the perpendicular line $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(4) - (-2)}{(1) - (7)} = \frac{4 + 2}{1 - 7} = \frac{6}{-6} = -1$ Since the two lines are perpendicular, then the product of their slopes equals to -1 . Hence, the slope of the line is 1. Now, we apply the equation $y - y_1 = m(x - x_1)$ $y - \left(-\frac{2}{3}\right) = (1)\left(x - \frac{1}{2}\right)$ $y + \frac{2}{3} = x - \frac{1}{2}$ $y = x - \frac{1}{2} - \frac{2}{3}$ $y = x - \frac{7}{6}$
40) Find the equation for the line passes through the point $\left(\frac{1}{2}, -\frac{2}{2}\right)$ and parallel to the	
through the point $\left(\frac{1}{2}, -\frac{1}{3}\right)$ and parallel to the line segment joining the points (1,4) and (7,-2). <u>Solution:</u> First, we find the slope of the parallel line $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(4) - (-2)}{(1) - (7)} = \frac{4 + 2}{1 - 7} = \frac{6}{-6} = -1$ Since the two lines are parallel, then they have the same slope. Now, we apply the equation $y - y_1 = m(x - x_1)$ $y - \left(-\frac{2}{3}\right) = (-1)\left(x - \frac{1}{2}\right)$ $y + \frac{2}{3} = -x + \frac{1}{2}$ $y = -x + \frac{1}{2} - \frac{2}{3}$ $y = -x - \frac{1}{6}$	